Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Tuesday 19 November 2019 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The number of messages, M, that six randomly selected teenagers sent during the month of October is shown in the following table. The table also shows the time, T, that they spent talking on their phone during the same month.

Time spent talking on their phone (T minutes)	50	55	105	128	155	200
Number of messages (M)	358	340	740	731	800	992

The relationship between the variables can be modelled by the regression equation $M = aT + b$.

(a) Write down the value of a and of b. [3]

(b) Use your regression equation to predict the number of messages sent by a teenager that spent 154 minutes talking on their phone in October. [3]

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2. [Maximum mark: 5]

Consider the lines L_1 and L_2 with respective equations

$$L_1 : y = -\frac{2}{3}x + 9 \text{ and } L_2 : y = \frac{2}{5}x - \frac{19}{5}.$$

(a) Find the point of intersection of L_1 and L_2. [2]

A third line, L_3, has gradient $-\frac{3}{4}$.

(b) Write down a direction vector for L_3. [1]

L_3 passes through the intersection of L_1 and L_2.

(c) Write down a vector equation for L_3. [2]

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3. [Maximum mark: 7]

Let $f(x) = x - 8$, $g(x) = x^4 - 3$ and $h(x) = f(g(x))$.

(a) Find $h(x)$. [2]

Let C be a point on the graph of h. The tangent to the graph of h at C is parallel to the graph of f.

(b) Find the x-coordinate of C. [5]

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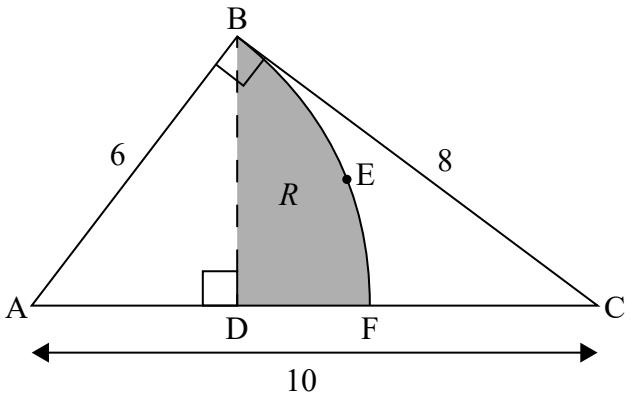
4. [Maximum mark: 7]

The following diagram shows a right-angled triangle, ABC, with $AC = 10\,\text{cm}$, $AB = 6\,\text{cm}$ and $BC = 8\,\text{cm}$.

The points D and F lie on [AC].
[BD] is perpendicular to [AC].
BEF is the arc of a circle, centred at A.
The region R is bounded by [BD], [DF] and arc BEF.

diagram not to scale



(a) Find $B\hat{A}C$. [2]

(b) Find the area of R. [5]

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5. [Maximum mark: 7]

The first two terms of a geometric sequence are $u_1 = 2.1$ and $u_2 = 2.226$.

(a) Find the value of r. [2]

(b) Find the value of u_{10}. [2]

(c) Find the least value of n such that $S_n > 5543$. [3]

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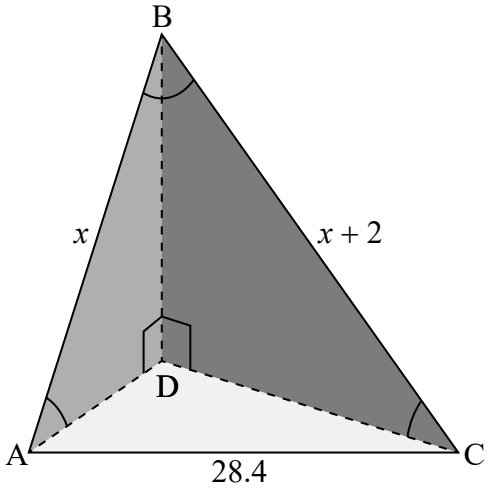
6. [Maximum mark: 6]

The diagram below shows a triangular-based pyramid with base ADC.
Edge BD is perpendicular to the edges AD and CD.

diagram not to scale



$AC = 28.4\,\text{cm},\ AB = x\,\text{cm},\ BC = x + 2\,\text{cm},\ A\hat{B}C = 0.667,\ B\hat{A}D = 0.611$

Calculate AD.

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7. [Maximum mark: 7]

The following table shows the probability distribution of a discrete random variable X, where $a \geq 0$ and $b \geq 0$.

x	1	4	a	$a + b - 0.5$
$\mathrm{P}(X = x)$	0.2	0.5	b	a

(a) Show that $b = 0.3 - a$. [1]

(b) Find the difference between the greatest possible expected value and the least possible expected value. [6]

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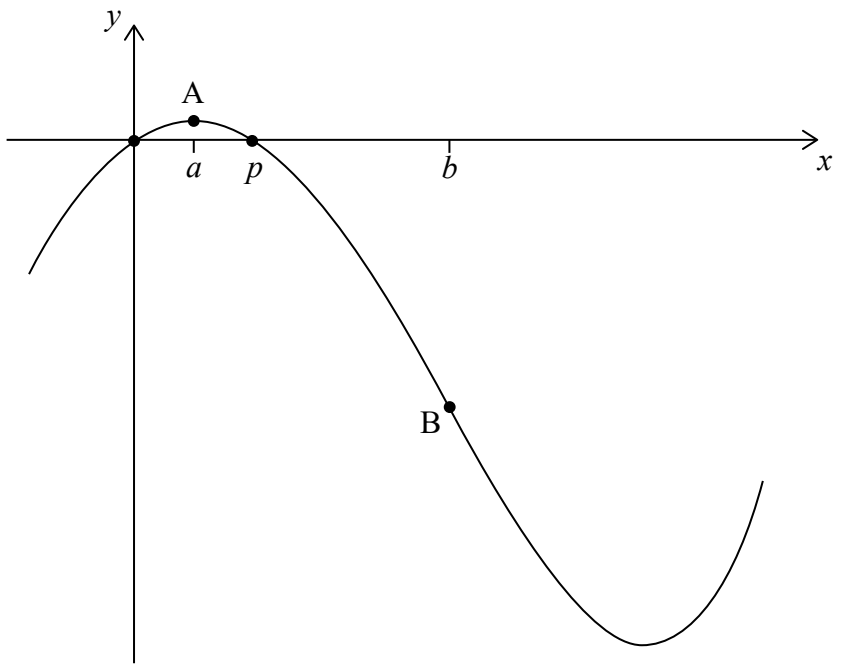
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $f(x) = x^4 - 54x^2 + 60x$, for $-1 \leq x \leq 6$. The following diagram shows the graph of f.



There are x-intercepts at $x = 0$ and at $x = p$. There is a maximum at point A where $x = a$, and a point of inflexion at point B where $x = b$.

(a) Find the value of p. [2]

(b) (i) Write down the coordinates of A.

(ii) Find the equation of the tangent to the graph of f at A. [4]

(c) (i) Find the coordinates of B.

(ii) Find the rate of change of f at B. [7]

(d) Let R be the region enclosed by the graph of f, the x-axis and the lines $x = p$ and $x = b$. The region R is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

SpeedWay airline flies from city A to city B. The flight time is normally distributed with a mean of 260 minutes and a standard deviation of 15 minutes.

A flight is considered late if it takes longer than 275 minutes.

(a) Calculate the probability a flight is **not** late. [2]

The flight is considered to be **on time** if it takes between m and 275 minutes. The probability that a flight is on time is 0.830.

(b) Find the value of m. [3]

During a week, SpeedWay has 12 flights from city A to city B. The time taken for any flight is independent of the time taken by any other flight.

(c) (i) Calculate the probability that at least 7 of these flights are **on time**.

(ii) Given that at least 7 of these flights are on time, find the probability that exactly 10 flights are on time. [7]

SpeedWay increases the number of flights from city A to city B to 20 flights each week, and improves their efficiency so that more flights are on time. The probability that at least 19 flights are on time is 0.788.

(d) A flight is chosen at random. Calculate the probability that it is on time. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

A rocket is travelling in a straight line, with an initial velocity of $140\,\text{m s}^{-1}$. It accelerates to a new velocity of $500\,\text{m s}^{-1}$ in two stages.

During the first stage its acceleration, $a\,\text{m s}^{-2}$, after t seconds is given by $a(t) = 240\sin(2t)$, where $0 \le t \le k$.

(a) Find an expression for the velocity, $v\,\text{m s}^{-1}$, of the rocket during the first stage. [4]

The first stage continues for k seconds until the velocity of the rocket reaches $375\,\text{m s}^{-1}$.

(b) Find the distance that the rocket travels during the first stage. [4]

During the second stage, the rocket accelerates at a constant rate. The distance which the rocket travels during the second stage is the same as the distance it travels during the first stage.

(c) Find the total time taken for the two stages. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.